WEB EXTENSION 7A

Derivation of Valuation Equations

The derivation of the formulas for the values of a perpetual preferred stock and the values of a constant growth stock are presented in this extension.

Values of a Perpetual Preferred Stock

The value of a perpetual preferred stock is given by

$$V_{ps} = \frac{D_{ps}}{(1+r_{ps})^1} + \frac{D_{ps}}{(1+r_{ps})^2} + \cdots + \frac{D_{ps}}{(1+r_{ps})^\infty} \quad \text{(7A-1)}$$

For a partial sum up to N, Equation 7A-1 may be rewritten as follows:

$$V_{ps} = D_{ps}\left[\frac{1}{(1+r_{ps})^1} + \frac{1}{(1+r_{ps})^2} + \cdots + \frac{1}{(1+r_{ps})^N}\right] \quad \text{(7A-2)}$$

Multiply both sides of Equation 7A-2 by $(1 + r_{ps})$, which yields

$$V_{ps}(1+r_{ps}) = D_{ps}\left[1 + \frac{1}{(1+r_{ps})^1} + \frac{1}{(1+r_{ps})^2} + \cdots + \frac{1}{(1+r_{ps})^{N-1}}\right] \quad \text{(7A-3)}$$

Subtract Equation 7A-2 from Equation 7A-3 to obtain

$$V_{ps}(1+r_{ps}-1) = D_{ps}\left[1 - \frac{1}{(1+r_{ps})^N}\right] \quad \text{(7A-4)}$$

As $N \to \infty$, $1/(1 + r_{ps})^N \to 0$ and Equation 7A-4 approaches

$$V_{ps}(r_{ps}) = D_{ps}$$

Thus we obtain Equation 7-8 from the chapter:

$$V_{ps} = \frac{D_{ps}}{r_{ps}}$$

Although we used preferred stock notation, Equations 7-3 and 7-8 are valid for any perpetuity.

VALUES OF A CONSTANT GROWTH STOCK

The proof of Equation 7-2, the chapter's formula for the value of a constant growth stock, $\hat{P}_0 = D_1/(r_s - g)$, is developed as follows. Rewrite Equation 7A-1 as

$$\begin{aligned}\hat{P}_0 &= \frac{D_0(1+g)^1}{(1+r_s)^1} + \frac{D_0(1+g)^2}{(1+r_s)^2} + \frac{D_0(1+g)^3}{(1+r_s)^3} + \cdots + \frac{D_0(1+g)^N}{(1+r_s)^N} \\ &= D_0\left[\frac{(1+g)^1}{(1+r_s)^1} + \frac{(1+g)^2}{(1+r_s)^2} + \frac{(1+g)^3}{(1+r_s)^3} + \cdots + \frac{(1+g)^N}{(1+r_s)^N}\right]\end{aligned} \quad \text{(7A-5)}$$

Multiply both sides of Equation 7A-5 by $(1 + r_s)/(1 + g)$:

$$\left[\frac{(1+r_s)}{(1+g)}\right]\hat{P}_0 = D_0\left[1 + \frac{(1+g)^1}{(1+r_s)^1} + \frac{(1+g)^2}{(1+r_s)^2} + \cdots + \frac{(1+g)^{N-1}}{(1+r_s)^{N-1}}\right] \quad \text{(7A-6)}$$

Subtracting Equation 7A-5 from Equation 7A-6, we obtain

$$\left[\frac{(1+r_s)}{(1+g)} - 1\right]\hat{P}_0 = D_0\left[1 - \frac{(1+g)^N}{(1+r_s)^N}\right]$$

Assume that $r_s > g$. Then, as N approaches infinity, the term in brackets on the right-hand side of the equation approaches unity, leaving

$$\left[\frac{(1+r_s)}{(1+g)} - 1\right]\hat{P}_0 = D_0$$

This can be rearranged as

$$\left[\frac{(1+r_s) - (1+g)}{(1+g)}\right]\hat{P}_0 = D_0$$

which simplifies to Equation 7-2 in the chapter:

$$\begin{aligned}(r_s - g)\hat{P}_0 &= D_0(1+g) = D_1 \\ \hat{P}_0 &= \frac{D_1}{r_s - g}\end{aligned}$$