The derivation of the formulas for the values of a perpetual preferred stock and the values of a constant growth stock are presented in this extension.

VALUES OF A PERPETUAL PREFERRED STOCK

The value of a perpetual preferred stock is given by

$$V_{ps} = D_{ps} + \frac{D_{ps}}{(1 + r_{ps})^2} + \frac{D_{ps}}{(1 + r_{ps})^3} + \ldots$$

(7A-1)

For a partial sum up to N, Equation 7A-1 may be rewritten as follows:

$$V_{ps} = D_{ps} \left[ 1 + \frac{1}{(1 + r_{ps})} + \frac{1}{(1 + r_{ps})^2} + \ldots + \frac{1}{(1 + r_{ps})^N} \right]$$

(7A-2)

Multiply both sides of Equation 7A-2 by $(1 + r_{ps})$, which yields

$$V_{ps}(1 + r_{ps}) = D_{ps} \left[ 1 + \frac{1}{(1 + r_{ps})} + \frac{1}{(1 + r_{ps})^2} + \ldots + \frac{1}{(1 + r_{ps})^N - 1} \right]$$

(7A-3)

Subtract Equation 7A-2 from Equation 7A-3 to obtain

$$V_{ps}(1 + r_{ps} - 1) = D_{ps} \left[ 1 - \frac{1}{(1 + r_{ps})^N} \right]$$

(7A-4)

As $N \to \infty$, $1/(1 + r_{ps})^N \to 0$ and Equation 7A-4 approaches

$$V_{ps}(r_{ps}) = D_{ps}$$

Thus we obtain Equation 7-8 from the chapter:

$$V_{ps} = \frac{D_{ps}}{r_{ps}}$$
Although we used preferred stock notation, Equations 7-3 and 7-8 are valid for any perpetuity.

**VALUES OF A CONSTANT GROWTH STOCK**

The proof of Equation 7-2, the chapter’s formula for the value of a constant growth stock, \( P_0 = \frac{D_1}{r_s - g} \), is developed as follows. Rewrite Equation 7A-1 as

\[
\hat{P}_0 = \frac{D_0(1 + g)}{(1 + r_s)} + \frac{D_0(1 + g)^2}{(1 + r_s)^2} + \frac{D_0(1 + g)^3}{(1 + r_s)^3} + \cdots + \frac{D_0(1 + g)^N}{(1 + r_s)^N}
\]

Multiply both sides of Equation 7A-5 by \((1 + r_s)/(1 + g)\):

\[
\frac{1}{(1 + g)} \left[ \frac{(1 + r_s)}{(1 + g)} \right] \hat{P}_0 = D_0 \left[ 1 + \frac{(1 + g)}{(1 + r_s)} + \frac{(1 + g)^2}{(1 + r_s)^2} + \cdots + \frac{(1 + g)^N}{(1 + r_s)^N} \right]
\]

(7A-6)

Subtracting Equation 7A-5 from Equation 7A-6, we obtain

\[
\frac{1}{(1 + g)} \left[ \frac{(1 + r_s)}{(1 + g)} \right] \hat{P}_0 = D_0 \left[ 1 - \frac{(1 + g)^N}{(1 + r_s)^N} \right]
\]

Assume that \( r_s > g \). Then, as \( N \) approaches infinity, the term in brackets on the right-hand side of the equation approaches unity, leaving

\[
\frac{1}{(1 + g)} \left[ \frac{(1 + r_s)}{(1 + g)} \right] \hat{P}_0 = D_0
\]

This can be rearranged as

\[
\frac{1}{(1 + g)} \left[ \frac{(1 + r_s) - (1 + g)}{(1 + g)} \right] \hat{P}_0 = D_0
\]

which simplifies to Equation 7-2 in the chapter:

\[
(r_s - g) \hat{P}_0 = D_0(1 + g) = D_1
\]

\[
\hat{P}_0 = \frac{D_1}{r_s - g}
\]